HW \#1 (Nonlinear Programming)

I. Solve Problems No:

4 and 6 from the following REVIEW QUESTIONS.

Due Date: Sunday 26/4/1436 H (15-2-2015) at the time of Tutorial (1:00 pm)
II. Solve Problems No:

1, 8 and 11 from the following REVIEW QUESTIONS.

Due Date: Thursday 30/4/1436 H (19-2-2015)
To be submitted before $12: 30 \mathrm{pm}$ at office (70-C / building 7)

Groups: The HW could be submitted in Groups, the maximum number of students for one group is 4 students. Groups can be formatted with less than 4 students.

1. Solve the following problem using a graphical method :

$$
\begin{aligned}
& f(x)=56.6 \mathrm{x}_{2} \sqrt{1+x_{1}^{2}} \\
& \frac{\left(1+x_{1}\right) \sqrt{\left(1+x_{1}^{2}\right)}}{2 \sqrt{2} x_{1} x_{2}} \leq 2, \\
& \frac{\left(x_{1}-1\right) \sqrt{\left(1+x_{1}^{2}\right)}}{2 \sqrt{2} x_{1} x_{2}} \leq 2,
\end{aligned}
$$

$0.1 \leq x_{1} \leq 2.0$,
$0.1 \leq x_{2} \leq 2.5$.
Solution: $\mathrm{X}^{*}=(0.6567,0.5335)$
$f(\mathrm{X})_{\text {min }}=36.127$
2. Determine the solution of the problem using a graphical procedure.

$$
\text { Minimize } f\left(x_{1}, x_{2}\right)=0.1 x_{1}+0.05773 x_{2}
$$

subject to

$$
\frac{0.6}{x_{1}}+\frac{0.3464}{x_{2}}-0.1 \leq 0, \quad 6-x_{1} \leq 0, \quad 7-x_{2} \leq 0
$$

3. Solve the following problem graphically:

Minimize $f\left(x_{1}, x_{2}\right)=\left(x_{1}-1\right)^{2}+x_{2}^{2}$
Subject to:

$$
\begin{aligned}
& g_{1}\left(x_{1}, x_{2}\right)=x_{1}^{3}-2 x_{2} \leq 0 \\
& g_{2}\left(x_{1}, x_{2}\right)=x_{1}^{3}+2 x_{2} \leq 0
\end{aligned}
$$

Solution: $(0,0)$
4. Find the maxima and minima, if any, of the function

$$
f(x)=4 x^{3}-18 x^{2}+27 x-7
$$

Solution:
$x^{*}=1.5$ (inflection point)
5. Find the minimum of the function

$$
f(x)=10 x^{6}-48 x^{5}+15 x^{4}+200 x^{3}-120 x^{2}-480 x+100
$$

Solution:
Hint: Use a computer solver to find the roots of an equation.
Solution: $x=-1$ (neither maximum nor minimum), $x=2$ (minimum).
6. Find the stationary (critical) points of the function:
$f(X)=\frac{1}{2} x_{1}^{2}+\frac{1}{2}\left(x_{2}-x_{1}\right)^{2}+\frac{1}{2} x_{2}^{2}-x_{2}$
Solution:
$\left(\frac{1}{3}, \frac{2}{3}\right)$ is a local minimum point.
7. Find the stationary (critical) points of the function:

$$
f(x, y)=x^{3}+x^{2} y-y^{2}-4 y
$$

Solution:
$(0,-2)$ local maximum, $\left(1,-\frac{3}{2}\right)$ saddle point, and $(-4,6)$ saddle point.
8. Solve the following NLP problem using the method of direct substitution:
$\operatorname{Max} z=4 x_{1}-0.14 x_{1}^{2}-0.1 x_{1}^{2}+5 x_{2}-0.2 x_{2}^{2}$
Subject to: $x_{1}+2 x_{2}=40$
Solution: $\mathrm{x}_{1}{ }^{*}=18.4, \mathrm{x}_{2}{ }^{*}=10.8, \mathrm{z}_{\text {max }}=70.42$
9. Solve the following NLP problem using the method of direct substitution:
$\operatorname{Max} z=\sqrt{x}+\sqrt{y}$
Subject to: $y=20-2 x$
Solution: $x^{*}=3.33, y^{*}=13.33, \mathrm{z}_{\max }=5.476$
10. Find the dimensions of a cylindrical tin (علبة صفيح دائرية) (with top and bottom) made up of sheet metal to maximize its volume such that the total surface area is equal to $A_{0}=24 \pi$. Use the Lagrange multipliers method.

Solution:

$$
\overline{x_{1}^{*}=2}, \quad x_{2}^{*}=4, \quad \lambda^{*}=-1, \quad \text { and } f^{*}=16 \pi
$$

11. Find the maximum of the function $f(\mathrm{X})=2 x_{1}+x_{2}+10$ subject to $g(X)=x_{1}+2 x_{2}^{2}=3$ using the Lagrange multipliers method.

## Solution:

$\mathbf{X}^{*}=\left\{\begin{array}{l}x_{1}^{*} \\ x_{2}^{*}\end{array}\right\}=\left\{\begin{array}{l}2.97 \\ 0.13\end{array}\right\} \quad \lambda^{*}=2.0$

## QUESTIONS FROM THE BOOK

10-28 Motorcross of Wisconsin produces two models of snowmobiles, the XJ6 and the XJ8. In any given production-planning week Motorcross has 40 hours available in its final testing bay. Each XJ6 requires 1 hour to test and each XJ8 takes 2 hours. The revenue (in $\$ 1,000 \mathrm{~s}$ ) for the firm is nonlinear and is stated as (Number of XJ6s)(4-0.1 number of XJ6s) + (Number of XJ8s)(5-0.2 number of XJ8s).
(a) Formulate this problem.
(b) Solve using Excel.

10-29 During the busiest season of the year, Green-Gro Fertilizer produces two types of fertilizers. The standard type ( X ) is just fertilizer, and the other type (Y) is a special fertilizer and weed-killer قاتل للعثب الضـار combination.
The following model has been developed to determine how much of each type should be produced to maximize profit subject to a labor constraint:
Maximize profit $=12 \mathrm{X}-0.04 \mathrm{X}^{2}+15 \mathrm{Y}-0.06 \mathrm{Y}^{2}$
subject to $2 \mathrm{X}+4 \mathrm{Y} \leq 160$ hours
$\mathrm{X}, \mathrm{Y} \geq 0$
Find the optimal solution to this problem using Excel.
10-30 Pat McCormack, a financial advisor for Investors RUs, is evaluating two stocks in a particular industry. He wants to minimize the variance of a portfolio consisting of these two stocks, but he wants to have an expected return of at least $9 \%$. After obtaining historical data on the variance and returns, he develops the following nonlinear program:
Minimize portfolio variance $=0.16 \mathrm{X}^{2}+0.2 \mathrm{XY}+0.09 \mathrm{Y}^{2}$
subject to: $\mathrm{X}+\mathrm{Y}=1$ (all funds must be invested)
$0.11 \mathrm{X}+0.08 \mathrm{Y} \geq 0.09$ (return on the investment)
$\mathrm{X}, \mathrm{Y} \geq 0$
Where:
$\mathrm{X}=$ proportion of money invested in stock 1
$\mathrm{Y}=$ proportion of money invested in stock 2
Solve this using Excel and determine how much to invest in each of the two stocks. What is the return for this portfolio? What is the variance of this portfolio?

10-31 Summertime Tees sells two very popular styles of embroidered مطرزة shirts in southern Florida: a tank top حمالات and a regular T-shirt. The cost of the tank top is $\$ 6$, and the cost of the T-shirt is $\$ 8$. The demand for these is sensitive to the price, and historical data indicate that the weekly demands are given by:
$\mathrm{X}_{1}=500-12 \mathrm{P}_{1}$
$\mathrm{X}_{2}=400-15 \mathrm{P}_{2}$
Where:
$X_{1}=$ demand for tank top
$\mathrm{P}_{1}=$ price for tank top
$\mathrm{X}_{2}=$ demand for regular T-shirt.
$\mathrm{P}_{2}=$ price for regular T-shirt
(a) Develop an equation for the total profit.
(b) Use Excel to find the optimal solution to the following nonlinear program.

Use the profit function developed in part (a).
Maximize profit
subject to $\mathrm{X}_{1}=500-12 \mathrm{P}_{1}$
$\mathrm{X}_{2}=400-15 \mathrm{P}_{2}$
$\mathrm{P}_{1} \leq 20$
$\mathrm{P}_{2} \leq 25$
$\mathrm{X}_{1}, \mathrm{P}_{1}, \mathrm{X}_{2}, \mathrm{P}_{2} \geq 0$

